Here is a list of the simulation functions defined in `simulations.xls`. Arguments in square brackets `[]` are optional; if omitted, they take the specified default value. If "Toggle random" is "off," the function returns the specified value (typically the mean). Valid parameter values are given; invalid values result in a #NUM error.

Continuous distributions:

`RandBeta(`*`mean`*`, `*`SD`*`, [`*`low`*`=0], [`*`high`*`=1])`

A beta distribution with a given ***mean*** and standard deviation ***SD***, optionally rescaled from ***low*** to ***high***.

Possible values: (***low***, ***high***)
Return value: ***mean***
Parameter values: $\mathit{low} < \mathit{mean} < \mathit{high}$,
$0 < \mathit{SD}$ and $\mathit{SD}^2 < (\mathit{mean} - \mathit{low})(\mathit{low} + 1 - \mathit{mean})$

`RandBetaAB(`*`alpha`*`, `*`beta`*`, [`*`low`*`=0], [`*`high`*`=1])`

A beta distribution with parameters ***alpha*** (α) and ***beta*** (β), optionally rescaled, which then has mean $= \mathit{low} + \frac{\alpha}{\alpha+\beta}$ and variance $= \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}$.

Possible values: (***low***, ***high***)
Return value: mean $= \mathit{low} + \frac{\alpha}{\alpha+\beta}$
Parameter values: $\mathit{alpha} > 0$, $\mathit{beta} > 0$, and $\mathit{low} < \mathit{high}$

`RandCauchy([`*`center`*`=0], [`*`scale`*`=1])`

A Cauchy distribution with the specified ***center*** and ***scale*** parameters.

Possible values: $(-\infty, +\infty)$
Return value: ***center***
Parameter values: $\mathit{scale} > 0$ (but no error is returned if $\mathit{scale} \leq 0$)

`RandExponential(`*`mean`*`)`

An exponential distribution with the specified ***mean***.

Possible values: $(0, +\infty)$, or $(-\infty, 0)$ if $\mathit{mean} < 0$
Return value: ***mean***
Parameter values: $\mathit{mean} \neq 0$

`RandGamma(`*`mean`*`, `*`SD`*`)`

A gamma distribution with the specified *mean* and standard deviation *SD*.

Possible values: $(0, +\infty)$
Return value: *mean*
Parameter values: *mean* > 0 and *SD* > 0

`RandGammaAB(`*`alpha`*`, `*`beta`*`)`

A gamma distribution with parameters *alpha* (α) and *beta* (β), which has mean $= \alpha\beta$ and variance $= \alpha\beta^2$.

Possible values: $(0, +\infty)$
Return value: mean $= \alpha\beta$
Parameter values: *alpha* > 0 and *beta* > 0

`RandLaplace([`*`center`*`=0], [`*`spread`*`=1])`

A Laplace (double-exponential) distribution with specified *center* and *spread*.

Possible values: $(-\infty, +\infty)$
Return value: *center*
Parameter values: *spread* > 0 (but no error is returned if *spread* ≤ 0)

`RandLogistic([`*`center`*`=0], [`*`spread`*`=1])`

A logistic distribution with specified *center* and *spread*.

Possible values: $(-\infty, +\infty)$
Return value: *center*
Parameter values: *spread* > 0 (but no error is returned if *spread* ≤ 0)

`RandLogNormal(`*`mean`*`, `*`SD`*`, [`*`low`*`=0])`

A log-normal distribution with specified *mean* and *SD*, optionally offset by the amount *low*. The associated normal distribution has variance $\sigma^2 = \ln\left[\left(\frac{SD}{mean - low}\right)^2 + 1\right]$ and mean $\mu = \ln(mean - low) - \frac{1}{2}\sigma^2$.

Possible values: $(low, +\infty)$
Return value: *mean*
Parameter values: *mean* $>$ *low* and *SD* > 0

`RandNormal([`*mean*`=0], [`*SD*`=1])`

A normal distribution with specified *mean* and *SD*.

Possible values: $(-\infty, +\infty)$
Return value: *mean*
Parameter values: $SD > 0$ (but no error is returned if $SD \leq 0$)

`RandPareto([`*low*`=1], [`*alpha*`=1])`

A Pareto distribution with shape parameter *alpha*.

Possible values: $(low, +\infty)$
Return value: median $= low \times \sqrt[alpha]{2}$
Parameter values: $low > 0$ and $alpha > 0$

`RandRayleigh([`*sigma*`=1])`

A Rayleigh distribution with parameter *sigma*.

Possible values: $(0, +\infty)$
Return value: median $= sigma\sqrt{\ln 4}$
Parameter values: $sigma > 0$

`RandT(`*df*`)`

A t distribution with degrees of freedom *df*.

Possible values: $(-\infty, +\infty)$
Return value: 0
Parameter values: $df > 0$

`RandTriangular(`*low*`, `*high*`, [`*peak*`=`$\frac{1}{2}$`(`*low*`+`*high*`)])`

A triangular distribution ranging from *low* to *high*, with specified *peak* value.

Possible values: $(low, high)$
Return value: *peak*
Parameter values: $low < high$, and $low \leq peak \leq high$

`RandUniform([`*low*`=0], [`*high*`=1])`

A continuous uniform distribution.

Possible values: $(low, high)$
Return value: mean $= \frac{1}{2}(low + high)$
Parameter values: $low < high$ (but no error is returned for invalid values)

`RandWeibull(`*`lambda, k`*`)`

A Weibull distribution with parameters *lambda* (scale) and *k* (shape).

Possible values:	$(0, +\infty)$
Return value:	mean $= lambda \times \Gamma(1 + 1/k)$
Parameter values:	$lambda > 0$ and $k > 0$

Discrete distributions:

`RandBinomial(`*`trials, prob`*`)`

A binomial distribution—the number of successes in *trials* independent attempts, each with probability of success *prob*.

Possible values:	$\{0, 1, 2, \ldots, trials\}$
Return value:	mean $= trials \times prob$
Parameter values:	$trials > 0$ (an integer), $0 < prob < 1$

`RandGeometric(`*`prob`*`)`

A geometric distribution—the number of failures before the first success, where each independent attempt has probability of success *prob*.

Possible values:	$\{0, 1, 2, \ldots\}$
Return value:	mean $= \frac{1-prob}{prob}$
Parameter values:	$0 < prob < 1$

`RandNegBinomial(`*`prob, numSuccesses`*`)`

A negative binomial distribution—the number of failures before succeeding *numSuccesses* times, where each independent attempt has probability of success *prob*.

Possible values:	$\{0, 1, 2, \ldots\}$
Return value:	mean $= numSuccesses \times \frac{1-prob}{prob}$
Parameter values:	$0 < prob < 1$, $numSuccesses > 0$ (an integer)

`RandHypergeometric(`*`nPop, nSample, r`*`)`

A hypergeometric distribution—the number of “special” items in a sample of size *nSample* from a population of size *nPop*, or which *r* are special.

Possible values:	$\{\max(0, nSample + r - nPop), \ldots, \min(r, nSample)\}$
Return value:	mean $= \frac{nSample \times r}{nPop}$
Parameter values:	$0 < nSample < nPop$, $0 < r < nPop$, all integers

`RandPoisson(`*`mean`*`)`

A Poisson distribution with the specified ***mean***.

Possible values: $\{0, 1, 2, \ldots\}$
Return value: ***mean***
Parameter values: ***mean*** > 0

`RandDiscrete(`*`values`*`, [`*`weights`*`])`

A general discrete distribution, with the specified set of possible ***values***, either in the form of a brace-enclosed list, or a range of cells. The optional ***weights*** (assumed to be all equal if omitted) determine the probabilities associated with the possible values. For example, to simulate selecting a coin from a box containing a penny, nickel, dime, and quarter, use `RandDiscrete({1,5,10,25})`. To choose a random value from the numbers stored in cells A1:A5, using the weights in cells B1:B5, use `RandDiscrete(A1:A5,B1:B5)`.

Possible values: specified by ***values***
Return value: the (approximate) median of the distribution
Parameter values: any list of ***values*** (all numeric). If ***weights*** are given, all must be numeric, and there must be as many weights as there are possible values. In addition, no weights can be negative, and at least one weight must be positive. The weights need not add to 1.

`RandSum(`*`count`*`, `*`x`*`)`

(Experimental) Computes the sum of ***count*** terms, each using the expression in ***x***. Typically, ***x*** (and possibly also ***count***) would be random. ***count*** is an integer expression, while ***x*** should be either a cell reference (to a cell containing a numeric value) or a string containing a valid numeric formula. For example, `RandSum(5,"rand()")` would compute the sum of 5 random numbers returned by the Excel function RAND(). `RandSum(RandPoisson(10),A5)` would evaluate the formula in cell A5, summing the result a random (Poisson) number of times.

Possible values: depends on the arguments
Return value: depends on the arguments
Parameter values:

`RandProduct(`*`count, x`*`)`

(Experimental) See the description of `RandSum` above

Possible values:
Return value:
Parameter values:

Technical details: In the following, $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$ is the gamma function, $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$ is the beta function, and $\mu_k = E(X^k)$.

Uniform distribution (continuous) — $\mathcal{U}(a,b)$

Parameters:	$a < b$
Support:	$a < x < b$
Density:	$f(x) = \frac{1}{b-a}$
Moments:	$E(X) = \frac{1}{2}(a+b)$, $\mathrm{Var}(X) = \frac{1}{12}(b-a)^2$.

Triangular distribution — $\mathcal{Tri}(a,b,m)$

Parameters:	$a < b$, and m with $a \leq m \leq b$
Support:	$a < x < b$
Density:	$f(x)$ is a triangle, with $f(a) = f(b) = 0$ and maximum value $f(m) = \frac{2}{b-a}$
Moments:	$E(X) = \frac{1}{3}(a+b+m)$, $\mathrm{Var}(X) = \frac{1}{18}(a^2+b^2+m^2-ab-am-bm)$.

Beta distribution — $\mathcal{Beta}(\alpha,\beta)$

Parameters:	$\alpha > 0$ and $\beta > 0$
Support:	$0 < x < 1$
Density:	$f(x) = \frac{1}{B(\alpha,\beta)}x^{\alpha-1}(1-x)^{\beta-1}$
Moments:	Mean $\frac{\alpha}{\alpha+\beta}$, $\mathrm{Var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$, $\mu_k = \frac{B(\alpha+k,\beta)}{B(\alpha,\beta)}$.
Notes:	Can be translated and rescaled to fall in an arbitrary interval.

Gamma distribution — $\mathcal{Gamma}(\alpha,\beta)$

Parameters:	$\alpha > 0$ and $\beta > 0$
Support:	$x > 0$
Density:	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$
Moments:	$E(X) = \alpha\beta$, $\mathrm{Var}(X) = \alpha\beta^2$, $\mu_k = \frac{\beta^k\Gamma(\alpha+k)}{\Gamma(\alpha)}$

Exponential distribution — $\mathcal{Exp}(\mu)$

Parameters: $\mu > 0$

Support: $x > 0$

Density: $f(x) = \frac{1}{\mu} e^{-x/\mu}$

CDF: $F(x) = 1 - e^{-x/\mu}$, for $x > 0$

Moments: $E(X) = \mu$, $\mathrm{Var}(X) = \mu^2$, $\mu_k = k!\mu^k$.

Notes: $\mathcal{Exp}(\mu)$ is the same as $\mathcal{Gamma}(\alpha = 1, \beta = \mu)$.

Weibull distribution — $\mathcal{Weibull}(\lambda, b)$

Parameters: $\lambda > 0$, $b > 0$

Support: $x > 0$

Density: $f(x) = \frac{b}{\lambda^b} x^{b-1} e^{-(x/\lambda)^b}$

CDF: $F(x) = 1 - e^{-(x/\lambda)^b}$, for $x > 0$

Moments: $E(X) = \lambda\Gamma(1 + 1/b)$, $\mathrm{Var}(X) = \lambda^2\Gamma(1 + 2/b) - E(X)^2$, $\mu_k = \lambda^k\Gamma(1 + k/b)$.

Notes: If $X \sim \mathcal{Weibull}(\lambda, b)$, then $X^b \sim \mathcal{Exp}(\lambda^b)$.

Rayleigh distribution — $\mathcal{Rayleigh}(\sigma)$

Parameters: $\sigma > 0$

Support: $x > 0$

Density: $f(x) = \frac{1}{\sigma^2} x e^{-x^2/2\sigma^2}$

CDF: $F(x) = 1 - e^{-x^2/2\sigma^2}$, for $x > 0$

Moments: $E(X) = \sigma\sqrt{\pi/2}$, $\mathrm{Var}(X) = \frac{1}{2}(4 - \pi)\sigma^2$.

Notes: $\mathcal{Rayleigh}(\sigma)$ is the same as $\mathcal{Weibull}(\lambda = \sigma\sqrt{2}, b = 2)$.

Pareto distribution — $\mathcal{Pareto}(x_m, \alpha)$

Parameters: $x_m > 0$, $\alpha > 0$

Support: $x > x_m$

Density: $f(x) = \alpha x_m^\alpha x^{-\alpha-1}$

CDF: $F(x) = 1 - (x_m/x)^\alpha$, for $x > x_m$

Moments: $E(X) = \frac{\alpha x_m}{\alpha - 1}$, $\mathrm{Var}(X) = \frac{\alpha x_m^2}{(\alpha-1)^2(\alpha-2)}$, $\mu_k = \alpha x_m^k/(\alpha - k)$ for $k < \alpha$

Notes: If $X \sim \mathcal{Pareto}(x_m, \alpha)$, then $\ln(X/x_m) \sim \mathcal{Exp}(1/\alpha)$.

Log-logistic distribution — $\mathcal{LogLog}(\alpha, \beta)$

Parameters:	$\alpha > 0$, $\beta > 0$
Support:	$x > 0$
Density:	$f(x) = (\beta/\alpha)(x/\alpha)^{\beta-1}[1 + (x/\alpha)^{\beta}]^{-2}$
CDF:	$F(x) = [1 + (x/\alpha)^{-\beta}]^{-1}$
Moments:	If $B = \pi/\beta$, then $E(X) = \alpha B/\sin B$ if $\beta > 1$, and $\mathrm{Var}(X) = \alpha^2(2B/\sin 2B - B^2/\sin^2 B)$ if $\beta > 2$. For $k < \beta$, $\mu_k = \alpha^k kB/\sin kB$.
Notes:	Median α. If $X \sim \mathcal{LogLog}(\alpha, \beta)$, then $\ln X \sim \mathcal{Log}(\mu = \ln \alpha, s = 1/\beta)$.

Lognormal distribution — $\mathcal{LN}(\mu, \sigma)$

Parameters:	μ real, $\sigma > 0$
Support:	$x > 0$
Density:	$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)$
Moments:	$E(X) = \exp(\mu + \sigma^2/2)$, $\mathrm{Var}(X) = (e^{\sigma^2} - 1)\exp(2\mu + \sigma^2)$, $\mu_k = \exp(k\mu + k^2\sigma^2/2)$.
Notes:	If $X \sim \mathcal{LN}(\mu, \sigma)$, then $\ln X \sim \mathcal{N}(\mu, \sigma)$.

Normal distribution — $\mathcal{N}(\mu, \sigma)$

Parameters:	μ real, $\sigma > 0$
Support:	$(-\infty, \infty)$
Density:	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right)$
Moments:	$E(X) = \mu$, $\mathrm{Var}(X) = \sigma^2$.

Student's t distribution — $t(\nu)$

Parameters:	$\nu > 0$
Support:	$(-\infty, \infty)$
Density:	$f(x) = \frac{1}{\sqrt{\nu}B(\nu/2, 1/2)}\left(1 + \frac{x^2}{\nu}\right)^{-\frac{1}{2}(\nu+1)}$
Moments:	$E(X) = 0$ if $\nu > 1$, $\mathrm{Var}(X) = \frac{\nu}{\nu - 2}$ if $\nu > 2$.
Notes:	Can be translated to have peak elsewhere.

Cauchy distribution — $\mathcal{Cauchy}(m, \gamma)$

Parameters: m real, $\gamma > 0$

Support: $(-\infty, \infty)$

Density: $f(x) = \frac{1}{\pi\lambda}\left[1 + \left(\frac{x-m}{\lambda}\right)^2\right]^{-1}$

CDF: $F(x) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-m}{\gamma}\right)$

Moments: All are undefined.

Notes: If $X \sim \mathcal{Cauchy}(m, \gamma)$, then $(X - m)/\gamma \sim t(1)$.

Laplace distribution — $\mathcal{Laplace}(\mu, b)$

Parameters: μ real, $b > 0$

Support: $(-\infty, \infty)$

Density: $f(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$

Moments: $E(X) = \mu$, $\mathrm{Var}(X) = 2b^2$.

Notes: Sometimes called the double-exponential distribution.

Logistic distribution — $\mathcal{Log}(\mu, s)$

Parameters: μ real, $s > 0$

Support: $(-\infty, \infty)$

Density: $f(x) = \frac{1}{4s}\operatorname{sech}^2\left(\frac{x-\mu}{2s}\right) = \frac{z}{s(1+z)^2}$ where $z = \exp(-(x-\mu)/2)$

CDF: $F(x) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x-\mu}{2s}\right)$

Moments: $E(X) = \mu$, $\mathrm{Var}(X) = \frac{1}{3}\pi^2 s^2$.

Hyperbolic secant distribution — $\mathcal{HypSec}(\mu, \sigma)$

Parameters: μ real, $\sigma > 0$

Support: $(-\infty, \infty)$

Density: $f(x) = \frac{1}{2\sigma}\operatorname{sech}\left(\frac{\pi(x-\mu)}{2\sigma}\right)$

CDF: $F(x) = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\sinh\left(\frac{\pi(x-\mu)}{2\sigma}\right)\right] = \frac{2}{\pi}\arctan\left[\exp\left(\frac{\pi(x-\mu)}{2\sigma}\right)\right]$

Moments: $E(X) = \mu$, $\mathrm{Var}(X) = \sigma^2$.

Gumbel distribution — $\mathcal{Gumbel}(\mu, \beta)$

Parameters: μ real, $\beta > 0$

Support: $(-\infty, \infty)$

Density: $f(x) = \frac{1}{\beta}ze^{-z}$ where $z = \exp(-(x-\mu)/\beta)$

CDF: $F(x) = \exp(-\exp(-(x-\mu)/\beta))$

Moments: $E(X) = \mu + \gamma\beta$, where $\gamma \approx 0.5772$, and $\mathrm{Var}(X) = \frac{1}{6}\beta^2\pi^2$.